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J. Laskar and P. Robutel

Equipe Astronomie et Systèmes Dynamiques, Bureau des Longitudes,
77 Avenue Denfert-Rochereau, F75014 Paris, France

Service des Calculs et de Mécanique Céleste du Bureau des Longitudes,
77, avenue Denfert-Rochereau
75014 Paris

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77 Avenue Denfert-Rochereau, F75014 Paris, France

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E-mail : **laskar@cosme.polytechnique.fr**

Abstract

The global stability of the spin axis orientation of the planets in relation to orbital secular perturbations is investigated using the method of frequency analysis. It is found that during their history, the obliquities of all the terrestrial planets may have passed through large-scale chaotic states. The obliquity of Mars is still in a large chaotic region ranging between 0° and 60°. Mercury and Venus have been stablilized by tidal dissipation, while the Earth may have been stabilized by the capture of the Moon. We thus find that none of the obliquities of the terrestrial planets should be considered as primordial.

Introduction

The origin of the obliquities of the planets (i.e., the orientation of their spin axes) is an important problem, because if the obliquities are primordial, they provide strong dynamical constraints on the formation of the solar system[1–4]. We have investigated the global dynamics, under planetary secular perturbations, of the obliquities and precession rates of all major planets of the solar system. We demonstrate that none of the obliquities of the inner planets (Mercury, Venus, the Earth and Mars) may be considered as primordial. Each of these planets could have originated with nearly zero obliquity, in a prograde state. The chaotic behavior of their obliquity then may have driven them to their current state. Mercury and Venus most probably underwent very large changes of obliquity, from 0° to about 90°, before tidal dissipation ultimately drove them to their current obliquity. The present obliquity of the Earth may have been reached during a chaotic state before the capture of the Moon[5]. The obliquity of Mars is presently chaotic, with possible variations ranging between 0° and about 60°. The obliquities of the outer planets are stable, but may have undergone similar variations in an earlier stage of the formation of the solar system.

Precession and Obliquity

Let $A = B < C$ be the moments of inertia of a planet. We assume that the axis of rotation of the planet is also its axis of maximum moment of inertia. The spin angular momentum is $\mathbf{H} = H\hat{\mathbf{H}}$, with $\hat{\mathbf{H}}$ a unit vector, and $H = Km\mathcal{R}^2\nu = C\nu$, where K is a coefficient depending on the internal structure of the planet ($K = 2/5$ for a homogeneous sphere), $\mathcal{R}$ its equatorial radius, m its mass, and ν its spin rate. Let $\hat{\mathbf{r}}$ be the unit vector in the direction of the Sun, and r the distance to the Sun. The torque exerted by the Sun, limited to first order in $\mathcal{R}/r$, is

$$\mathbf{L} = \frac{3GM}{r^3}\hat{\mathbf{r}} \times I\hat{\mathbf{r}}, \tag{1}$$

where I is the inertia matrix[6], G the gravitational constant, and M the solar mass. The precession motion of the planet is given by $d\mathbf{H}/dt = \mathbf{L}$. By averaging over the time (i.e., over the mean anomaly), we obtain the equations of precession, that is the secular motion of the spin axis of the planet

$$\frac{d\hat{\mathbf{H}}}{dt} = \alpha(1-e^2)^{-3/2}(\hat{\mathbf{H}}\cdot\hat{\mathbf{Z}})\hat{\mathbf{H}} \times \hat{\mathbf{Z}}, \tag{2}$$

where $\hat{\mathbf{Z}}$ is the unit vector in the direction of orbital angular momentum, e the eccentricity, and where

$$\alpha = \frac{3GM}{2Ka^3\nu}\frac{C-A}{C},$$

with a the semi-major axis of the planet, will be called the precession constant. Let E_{c0} be the orbital plane at the time origin with equinox γ_0, E_{ct} the orbital plane of the date, with equinox γ (cf. Fig. 1), and N the intersection of E_{c0} and E_{ct}. The precession in longitude is $\psi = \Lambda - \Omega$, where $\Omega = \gamma_0 N$ is the longitude of the node, and $\Lambda = \gamma N$. Let γ_0' be the point of E_{ct} such that $\gamma_0' N = \Omega$. The coordinates of $\hat{\mathbf{H}}$ in the reference frame E_{ct} with origin γ_0' are $(\sin\psi\sin\varepsilon, \cos\psi\sin\varepsilon, \cos\varepsilon)$, where ε is the obliquity, that is, the angle between the equator (E_{qt}) and orbital plane of the date (E_{ct}). After some computation, and using the action variable $X = \cos\varepsilon$ and the notation $p = \sin(i/2)\sin(\Omega)$, $q = \sin(i/2)\cos(\Omega)$, ($i$ is the inclination of E_{ct} with respect to E_{c0}), we obtain the equations of precession

$$\frac{d\psi}{dt} = \frac{\partial \mathcal{H}}{\partial X}; \qquad \frac{dX}{dt} = -\frac{\partial \mathcal{H}}{\partial \psi}, \tag{3}$$

which are related to the Hamiltonian

$$\mathcal{H}(X,\psi,t) = \frac{\alpha}{2}\,(1-e(t)^2)^{-3/2}X^2 + \sqrt{1-X^2}(\mathbf{A}(t)\sin\psi + \mathbf{B}(t)\cos\psi)\ , \tag{4}$$

with

$$\begin{aligned}\mathbf{A}(t) &= 2\ \left(\dot{q} + p(q\dot{p} - p\dot{q})\right)/\sqrt{1-p^2-q^2}\\ \mathbf{B}(t) &= 2\ \left(\dot{p} - q(q\dot{p} - p\dot{q})\right)/\sqrt{1-p^2-q^2}\\ \mathbf{C}(t) &= (q\dot{p} - p\dot{q})\ .\end{aligned}$$

The expression $\mathbf{A}(t) + i\mathbf{B}(t)$ and the eccentricity $e(t)$ describe the orbital motion of the planet and will be given by Laskar's La90 solution[7]. Since the orbital motion of the planets—especially the inner planets—is chaotic[7,9–12], a quasiperiodic approximation of $\mathbf{A}(t) + i\mathbf{B}(t)$ is not well suited to obtaining accurate solutions over several million years. This is reflected by the appearance of the filtered Fourier spectrum of $\mathbf{A}(t)+i\mathbf{B}(t)$ for the inner planets (Fig. 2), where only the main peaks can be identified as combinations of the planetary secular fundamental frequencies. A quasiperiodic approximation of $\mathbf{A}(t)+i\mathbf{B}(t)$ of the form

$$\mathbf{A}(t) + i\mathbf{B}(t) \approx \sum_{k=1}^{N} \alpha_k e^{i(\nu_k t + \phi_k)}$$

may still be used to study the outer planets, or more important, for a qualitative understanding of the behavior of solutions. Indeed, with this approximation, the Hamiltonian reads

$$H = \frac{\alpha}{2}(1-e(t)^2)^{-3/2}X^2 + \sqrt{1-X^2}\sum_{k=1}^{N}\alpha_k \sin(\nu_k t + \psi + \phi_k)\ , \tag{5}$$

which is the Hamiltonian for an oscillator with frequency αX, perturbed by a quasiperiodic external oscillation with several frequencies ν_k. Resonance occurs when $\dot{\psi} \approx \alpha X = \alpha\cos\varepsilon$ is the negative of one of the frequencies ν_k. When limited to a single periodic term, and with $\epsilon = Cte$, this Hamiltonian is integrable and is often referred to as Colombo's top[13].

Its equilibrium points are then called Cassini states[14]. But in the present case, $e(t)$ and $\mathbf{A}(t)+i\mathbf{B}(t)$ take into account the secular perturbations of the whole solar system, modeled as a 15 degree of freedom system, and this Hamiltonian has $1+15$ degrees of freedom. Its solutions evolve in a phase space of dimension $2+30$ (2 arises from the ψ, X variables). In fact, the orbital solution La90 is not coupled with the precession variables, and will thus never change. The Hamiltonian is thus considered to be time-dependent, but in a non-periodic way. No traditional tools like Poincaré surface of sections can be used to analyze its dynamics, and we will rely on Laskar's method of frequency analysis[7,15,16].

Frequency Analysis

The method of numerical analysis of the fundamental frequencies was introduced in the study of the stability of the solar system[7]. In that case, frequency analysis permitted numerical estimates of the size of chaotic zones in all directions of the 15 degrees of freedom, and revealed that for the inner planets, the chaotic zone is relatively large, while for the outer planets (Jupiter to Neptune), this zone is much smaller. More generally, the frequency analysis method (which will only be outlined here), can be applied to study the stability of the solutions of a conservative dynamical system, and is based on a refined numerical search for quasiperiodic approximations of solutions over a finite time span[7,15,16].

If $f(t)$ is a function with values in the complex domain, obtained numerically over a finite time span $[-T, T]$ the frequency analysis algorithm consists in the search for a quasiperiodic approximation of $f(t)$ with a finite number of periodic terms of the form

$$\tilde{f}(t) = \sum_{k=1}^{N} a_n e^{i\sigma_k t} \; .$$

The frequencies σ_k and complex amplitudes a_k are found with an iterative scheme. To determine the first frequency σ_1, one searches for the maximum of the amplitude of $\phi(\sigma) = \langle f(t), e^{i\sigma t}\rangle$, where the scalar product $\langle f(t), g(t)\rangle$ is defined by

$$\langle f(t), g(t)\rangle = \frac{1}{2T}\int_{-T}^{T} f(t)\bar{g}(t)\chi(t)dt \; ,$$

and where $\chi(t)$ is a weight function, that is, a positive function satisfying $1/2T \int_{-T}^{T} \chi(t)dt = 1$. Once the first periodic term $e^{i\sigma_1 t}$ is found, its complex amplitude a_1 is obtained by orthogonal projection, and the process is repeated on the remaining part of the function $f_1(t) = f(t) - a_1 e^{i\sigma_1 t}$. A second analysis is also carried out in order to estimate the precision of the determination. In the case of an integrable Hamiltonian system with n degrees of freedom, the frequency analysis of the solutions will give their quasiperiodic expansion and in particular will determine the vector $(\nu_i)_{i=1,n}$ of the fundamental frequencies of the system.

In the case of nearly integrable systems, if an orbit is not regular (quasiperiodic), frequency analysis gives a quasiperiodic approximation to the solution which holds only locally in time. In other words, it gives a frequency vector $(\nu_i(t))_i$ for each value of t, obtained by applying the frequency analysis algorithm over the time span $[t, t+T]$. In analyzing the precession and obliquity, we shall only look for the precession frequency p which is associated to the angle and action variables ψ, X. The initial phase ψ_0 is fixed; if we fix also some initial conditions X, we can carry out the frequency analysis for the orbits corresponding to initial conditions (ψ_0, X) (at $t = 0$) over the time span $[0, T]$. We thus define the frequency map

$$F_T : \mathbf{R} \longrightarrow \mathbf{R}$$
$$X \longrightarrow p\ .$$

The regularity of the precession and obliquity can be analyzed in a precise way by studying the regularity of the frequency map. Indeed, a simple numerical criterion for the destruction of KAM curves obtained for 2 degree of freedom systems can be extended to this problem of $1+n$ degrees of freedom. This criterion ensures that invariant surfaces are destroyed as soon as the frequency map is no longer monotonic. Distorsions of the frequency map permit statements about the non-existence of tori, and as these distorsions increase, they produce a complete loss of regularity of the frequency map, which is also an indication of chaotic motion[16].

Methodology

If the motion of the solar system were quasiperiodic, the motion of precession could be modeled by Eqs. 3 and 5. One could then use Chirikov's resonance orverlap criterion[17] to get an idea of when the motion of precession becomes chaotic. This can be done for the outer planets, but in order to obtain more accurate results, we directly integrated the full equations of precession (3,4), and performed frequency analysis on the precession solutions. Since the dynamic ellipticity $(C - A)/C$ is proportional[18] to ν^2, the precession constant α is proportional to ν. The primordial value of α thus depends on the primordial rotation speed of the planets, which has to be estimated. We performed many numerical integrations of the precession equations for all values of the intial obliquity, every $0.1°$. These integrations use the orbital solution La90 as an input and are, in general, performed over 18Myr with a stepsize of about 200yr. We used frequency analysis to obtain the frequency curve, the regularity of which indicates the regularity of the precession and obliquity. For each integration, we retained the maximum and minimum values reached by the obliquity during the integration. From this global view of the dynamics of precession, it is also possible to understand what would happen for slightly different values of the precession constant α. This is important, since the present—and to a greater extent the primordial—values of this constant are not known accurately.

Results

Mercury. Mercury's present spin is very slow, and apparently trapped in a (2:3) spin-orbit resonance. The primordial spin rate of Mercury was probably much higher, and was slowed down by solar tides. Upon extrapolating the apparent correlation of angular momentum with mass[19], the primordial rotation period of Mercury is found to be 19 h[20]. This leads to a precession frequency of 127 $''$/yr. Due to tidal interactions with the Sun, this frequency then decreases[21,22]. We will only assume that the primordial period of rotation of Mercury was larger than 85 h, which gives a precession constant of 30 ″/yr. A plot of the precession frequency p against the initial obliquity ε_0 (Fig. 3a) shows that for small values of the obliquity the motion is stable, while there exists a very large chaotic zone ranging from 50° to 100°. When the rotation speed of the planet decreases, the precession constant α decreases proportionally. Even if the initial obliquity of Mercury is very small, for a precession constant of about 20 ″/yr, it enters a very large chaotic zone extending from 0° to about 100°. During this period, and until tidal dissipation ultimately drives Mercury into its present state, its orientation undergoes very large changes from 0° to 100° over a few million years (Fig. 3d). It can thus be said that during its history Mercury underwent large-scale chaotic behavior, at some time having a completely different orientation than it has now with its pole facing the Sun. This tumultuous history may have left some traces on the surface of the planet.

Venus. The orientation of the pole of Venus (178°) is one of the puzzling features of the solar system[1,24,25]. The atmospheric tides on the surface of Venus can explain possible changes in orientation of the spin from 90° to the present value, but not more[25]. It is thus generally assumed that the retrograde rotation of Venus is primordial, which is one of the major arguments supporting the stochastic accretion mechanism for the terrestrial planets[3,4].

Following the previous study of Mercury, we used frequency analysis to investigate the global dynamics of the precession and obliquity of Venus. According to Goldreich[20], the primordial rotation speed of Venus was about 13h, which gives a precession constant of 31 ″/yr. We began our investigation with a precession constant of 40 ″/yr, which reveals a regular behavior for small obliquities, and then a large chaotic region from 50° to 90°. Because of solar tidal dissipation, the planet rotation speed then decreases, and consequently so does the precession constant. When the precession constant reaches 20 ″/yr, which corresponds to a period of about 20h, the chaotic zone extends from 0° to nearly 90° (Fig. 4a,b). In this figure, we see several well defined chaotic zones, but the frequency analysis shows that no invariant surfaces bound the motion, so that orbits may diffuse in a few million years from 0° to nearly 90°. When the precession frequency decreases further, the shape of the chaotic region changes (Fig. 4c,d). Once the obliquity of Venus reaches about 90°, it could stabilize around that value, which lies outside the chaotic zone. As the rotation continues to slow, atmospheric tides might then drive the spin axis toward its present orientation.

We thus present a mechanism which can drive the obliquity of Venus from 0° to the present value of 178°, which shows that it is not necessary to consider its retrograde rotation as primordial. Moreover, during its history, Venus has most probably experienced large-scale chaotic changes in orientation, with its pole perhaps facing the Sun over extended periods. Such periods include the likelihood of strong changes in climate and atmospheric circulation.

Mars. Analysis of the dynamics of Mars' obliquity is more staightforward, as its rotation speed can be considered primordial. Indeed, Mars is far from the Sun and has no large satellite to slow it with tidal interactions. Ward[26] showed that, in contrast with the Earth, the obliquity of Mars suffers large variations of ±10° around its mean value of 25° because of planetary secular perturbations. The precession constant of Mars[27,28] (8.26 ″/yr) is close to some secular frequencies of the planet, and the question of Mars' possible passage through resonance has been raised several times[26,28], without definite conclusions due to the lack of knowledge of the precise intitial conditions.

In the present work, we investigated the problem in a very different manner, by looking at its global dynamics. We first carried out a frequency analysis of Mars' obliquity over 45 Myr. We found a large chaotic zone ranging from about 0° to 60°. Indeed, in Fig. 5a,b, the motion appears regular for small values of the initial obliquity. But the motion of the inner planets is chaotic[7,9], and the phases of the planetary secular terms are lost after about 100 Myr. We can thus study the problem independently of the phase, and Fig. 5c,d was produced with a small change of the initial date (≈-3Myr) of the orbital solution of Mars. This should convince the reader that the chaotic zone actually extends from nearly 0° to 60°. Indeed, Fig. 5d shows that some orbits display dramatic changes over time intervals of less than 45 Myr.

From this analysis, we conclude that the obliquity of Mars is chaotic, with possible variations ranging from nearly 0° to about 60°. We also computed the Liapounov exponent of Mars' obliquity for the present initial conditions[29] and found a value similar to those for the other planets[9], that is, about $1/5Myr$. We verified that this value reveals the intrinsic chaotic behavior of the obliquity of Mars by carrying out the computation again with a "regular" value (50 ″/yr) of the precession constant. Nevertheless, we consider the frequency analysis to be more significant, as it gives the extent of the chaotic zone. Moreover, since it gives a global picture of the dynamics, it shows that small changes in the initial conditions or in the model will not affect this result substantially.

The chaotic behavior of Mars' orientation might have been an important factor in its evolution, since its obliquity may have reached 60° during its history. The obliquity of Mars cannot be considered as primordial; the planet may have been formed with near zero obliquity and then driven to its present value only by the effect of secular planetary perturbations.

The Earth. The equations for the precession of the Earth are more complicated due to the presence of the Moon, and the problem of the stability of the Earth's obliquity was studied in previous papers[5,8]. Contrary to the conclusions of Ward's pioneering paper[30], we found that the present climatic stability of the Earth results from the presence of the Moon. Indeed, the variations of the Earth's obliquity are only ±1.3° around the mean value of 23.3° (Fig. 6b), but if the Moon had not been present, the torque exerted by the Sun on the Earth would have been smaller, and the precession constant of the Earth would be about 30 ″/yr for a rotation speed of 15 h. In this case, in a way similar to Venus and Mars, the Earth would be in a very large chaotic zone, extending from 0° to about 85° (Fig. 6c,d). This very likely would have prevented the appearance of organized life on the Earth. Although more work needs to be done on this problem, it should be emphasized that in a generic planetary system (i.e., a planetary system similar to ours), the probability of existence of a planet with climate stability similar to ours should be of the same order as the probability for a generic planet like the Earth to capture a large satellite like the Moon. This should decrease estimates of the probability of finding organized life in the vicinity of the solar system by several orders of magnitude.

As with Mars, we cannot consider the Earth's obliquity to be primordial. The Earth could have been formed with near zero obliquity, without the Moon. It would thus be in a large-scale chaotic region with rapid (on a million year time scale) variations from 0° to 50°, and a mean value around 25°. The capture of the Moon would then have frozen the obliquity at its current value of about 23°.

The outer planets. We also analyzed the global stability of the obliquity of the outer planets (Jupiter, Saturn, Uranus, Neptune). The orbital solutions of these planets behave very differently from those of the inner planets. As attested by the different spectra of $\mathbf{A}(t) + i\mathbf{B}(t)$, (Fig. 2), they display only well isolated lines. There is no large overlap of the different resonant terms as for the inner planets, and the solutions are essentially regular. Estimates of the precession constants for the outer planets are not very precise because their structure is not well known, but they can be well estimated[31,32] below 5 ″/yr. However, since we obtain a global picture of the dynamics, we also understand the behavior of the obliquities for slightly different values of the constants. Nothing can disturb their stability until the precession constants of these planets reaches about 26 ″/yr, so that resonance with s_6 occurs. Even then (this resonance being isolated), the motion will display large oscillations, but in a regular manner. We can therefore say that, as was the case for the orbital motion, the obliquities of the outer planets are essentially stable, and thus should be considered as primordial.

But here primordial means that the obliquities have not changed since the solar system was in the final stages of formation, in a similar state (at least as concerns the outer planets), as at present. In a previous stage of its formation, if it were more massive than now, instabilities similar to those of the inner planets might have occurred, which would have driven the obliquities of the outer planets to their present configurations, and we intend to further explore this hypothesis in the future.

Conclusions

None of the obliquities of the inner planets should be considered as primordial, and all of these planets could have been formed with near zero obliquity. The obliquities of these planets have probably undergone large-scale chaotic variation during their history. Mercury and Venus have been stabilized by dissipative effects, the Earth may have been stabilized by the capture of the Moon, and Mars is still presently in a large chaotic zone, ranging from 0° to 60°.

It should also be noted that planets in a retrograde spin state would be much more stable than planets with a prograde spin, as all the main forcing frequencies of the inclination are negative (Fig. 2).

The obliquities of the outer planets are essentially stable, and can thus be considered as primordial, that is, to have about the same values they had at the end of the formation of the solar system. Nevertheless, chaotic behavior of the obliquities under planetary perturbations could have occured in an earlier stage of the formation of the solar system, at a time when it could have been more massive, and this point should be clarified by further studies.

In a generic planetary system, a planet similar to the Earth would suffer large-scale chaotic changes in its obliquity unless it captured a massive satellite like the Moon. The climatic variations at its surface would then be very important, and most probably would prevent the appearance of organized life. This decreases by several orders of magnitude the present estimates of the probability of finding organized life in the vicinity of the solar system.

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Figure Captions

Fig 1. Fundamental planes for the definition of precession. Eq_t and Ec_t are the mean equator and ecliptic of the date. Ec_0 is the fixed J2000 ecliptic, with equinox γ_0. The general precession in longitude ψ is defined by $\psi = \Lambda - \Omega$. γ_0' is defined by $\gamma_0' N = \gamma_0 N = \Omega$, i is the inclination.

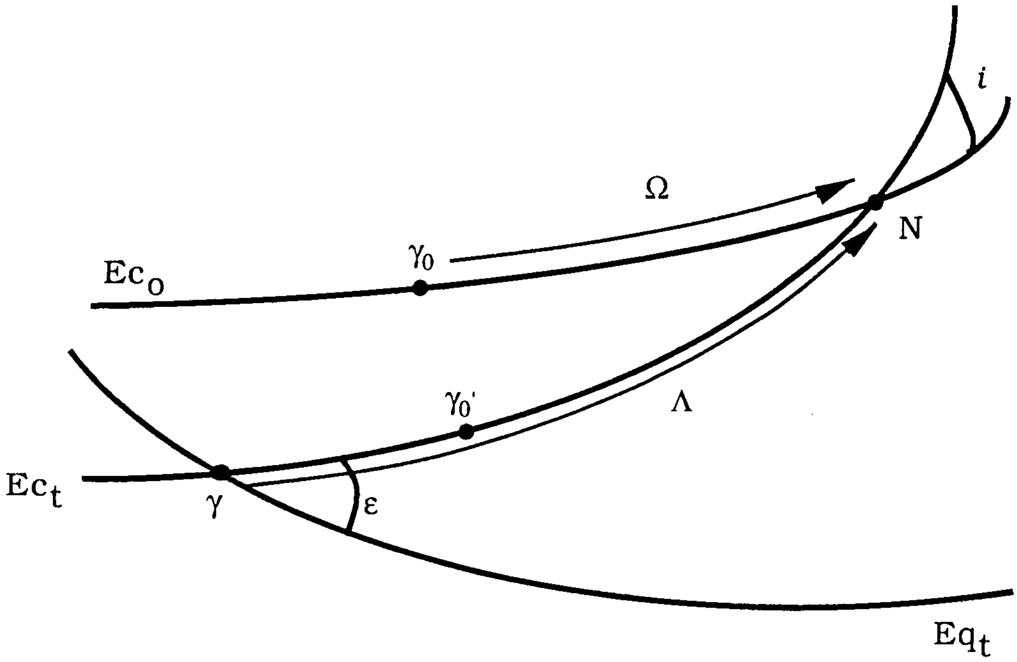
Fig 2. Fourier spectrum (with hanning filter) of the planetary forcing term in inclination $\mathbf{A}(t) + i\mathbf{B}(t)$. The logarithm of the amplitude of the Fourier coefficients are plotted against the frequency in $''$/yr.

Fig 3. Frequency analysis of the obliquity of Mercury over 18Myr for the values $\alpha = 30$ $''$/yr (a) and $\alpha = 10$ $''$/yr (b) of the precession constant, which correspond to rotation periods of about 85h and 255h. The precession frequency is plotted against the initial obliquity. The regularity of the frequency curve shows the regularity of the motion. When $\alpha = 10$ $''$/yr, the chaotic zone extends form $0°$ to $100°$. The maximum, mean, and minimum values of the obliquity reached over 18Myr are plotted in (b) and (d).

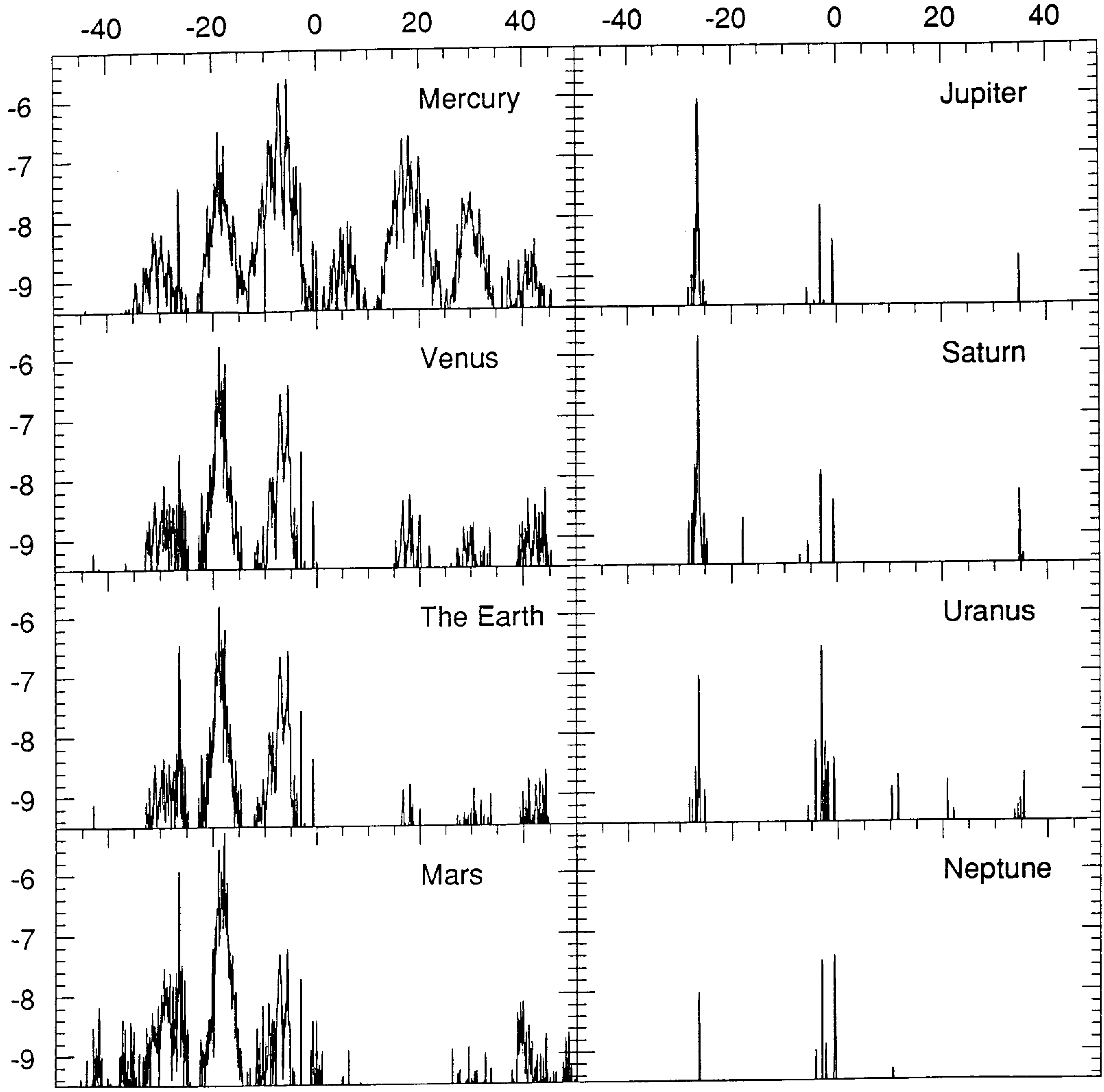
Fig 4. Frequency analysis of the obliquity of Venus over 18Myr for the values $\alpha = 20$ $''$/yr (a) and $\alpha = 10$ $''$/yr (b) of the precession constant, which correspond to rotation periods of about 20h and 40h. The precession frequency is plotted against the initial obliquity. The regularity of the frequency curve shows the regularity of the motion. In (a), the chaotic zone extends from $0°$ to nearly $90°$. Then, as the rotation speed decreases, Venus can be stabilized with obliquity around $90°$, before tidal interactions with the sun drive it to its current state. The maximum, mean, and minimum values of the obliquity reached over 18Myr are plotted in (b) and (d).

Fig 5. Frequency analysis of the obliquity of Mars over 45Myr. In (a) and (b), the planetary solution is used with the present initial conditions. A large chaotic zone is visible, ranging from $0°$ to $60°$. When the phase of the planetary solution is shifted by about -3Myr, this large chaotic zone is even more visible (c), (d). The maximum, mean, and minimum values of the obliquity reached over 45Myr are plotted in (b) and (d).

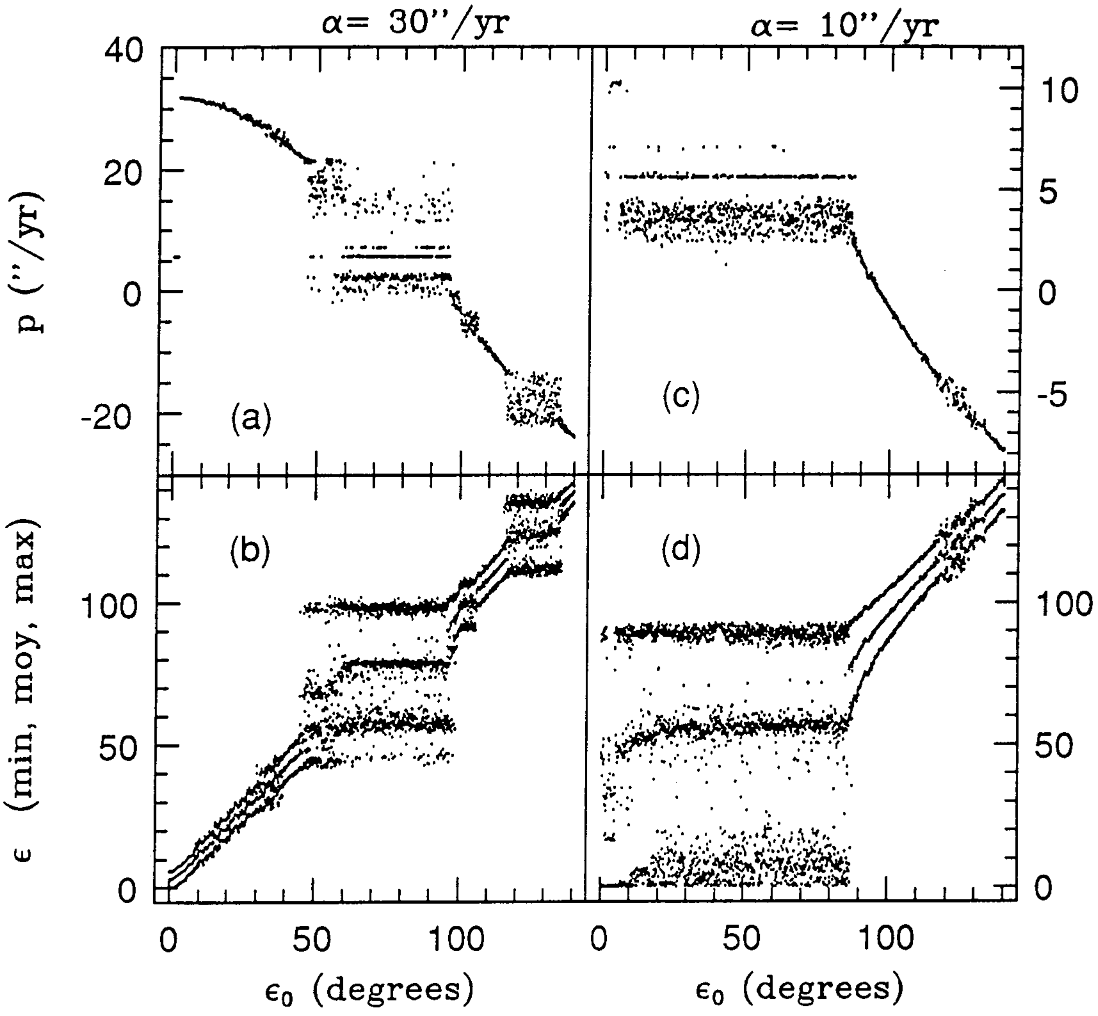
Fig 6. Frequency analysis of the obliquity of the Earth over 18Myr. The analysis of the precession frequency for all values of the obliquity reveals a large chaotic zone ranging from 50° to 90° for the present spin rate of the Earth, and in presence of the Moon. If the Moon did not exist, under the hypothesis of a 15h period for the Earth, the chaotic zone would extend from nearly 0° to about 90°. The maximum, mean, and minimum values of the obliquity reached over 18Myr are plotted in (b) and (d).



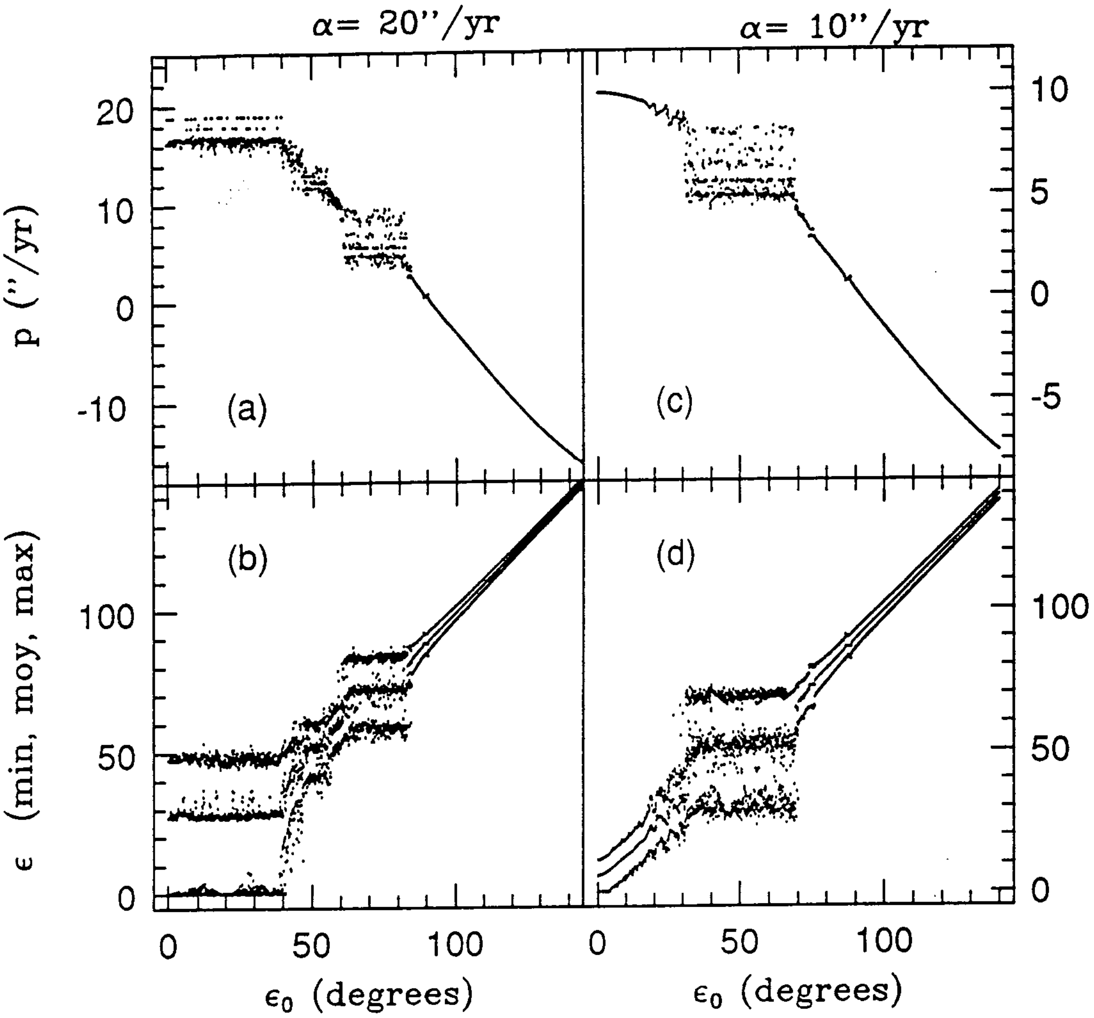
Laskar and Robutel Fig 1



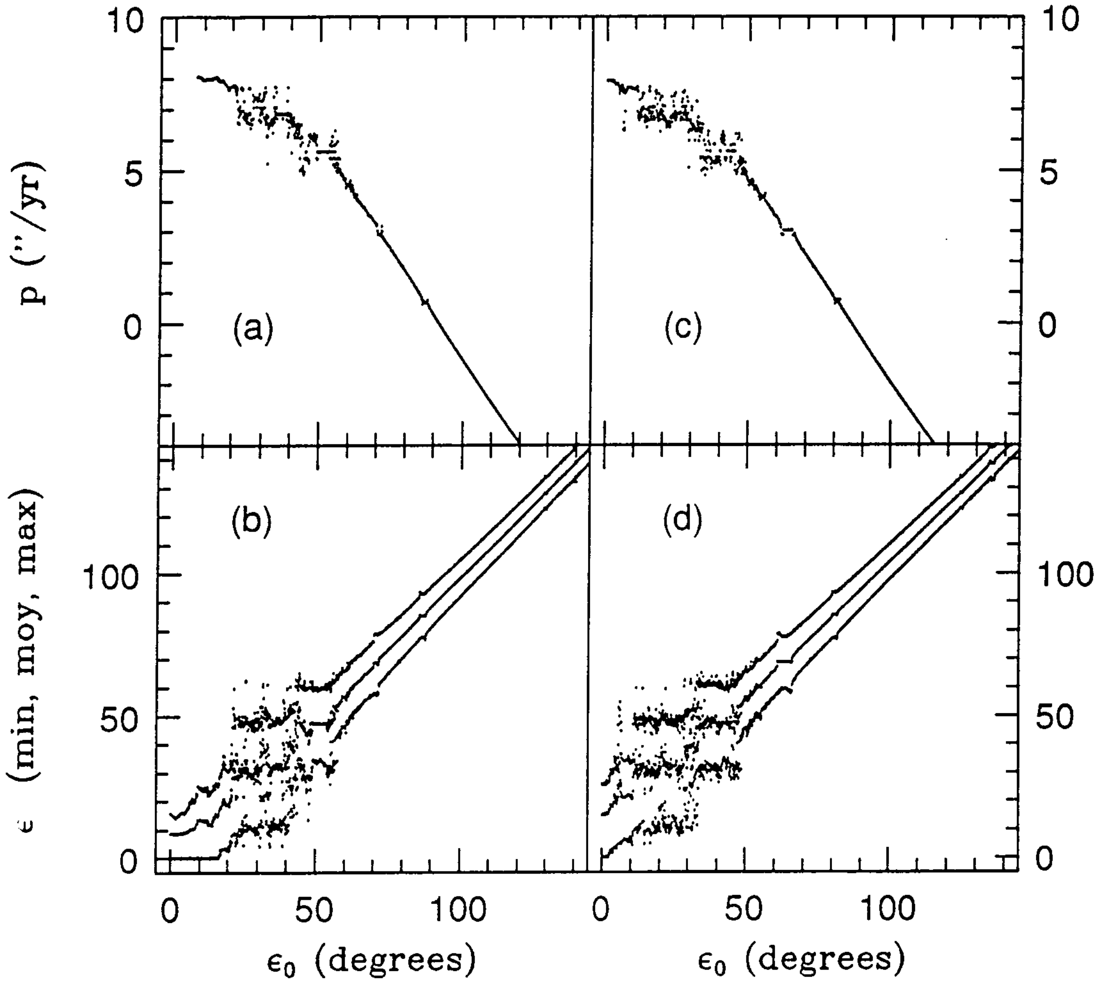
Laskar and Robutel Fig 2



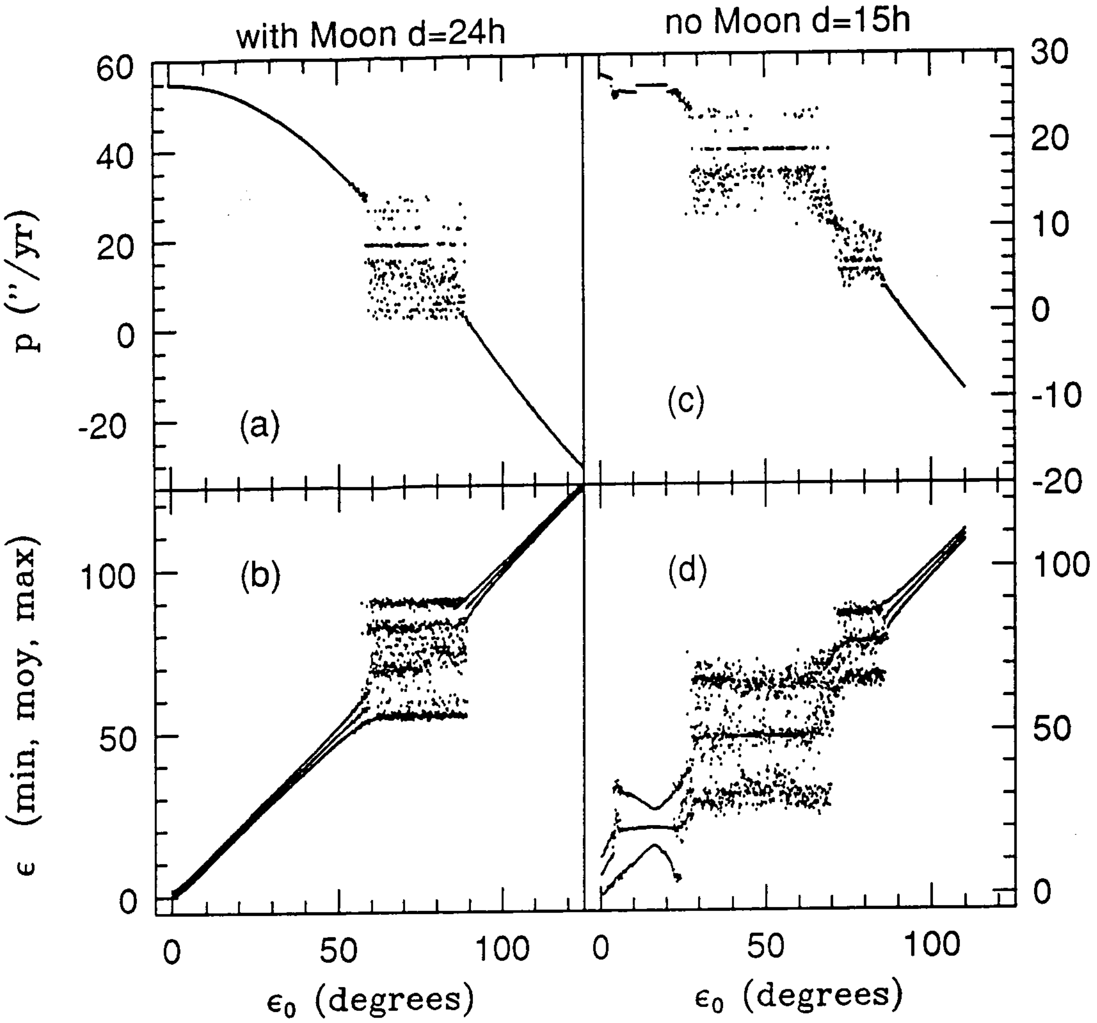
Laskar and Robutel Fig 3



Laskar and Robutel Fig 4



Laskar and Robutel Fig 5



Laskar and Robutel Fig 6